Tight any-shot quantum decoupling

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[arxiv : 2602.17430]

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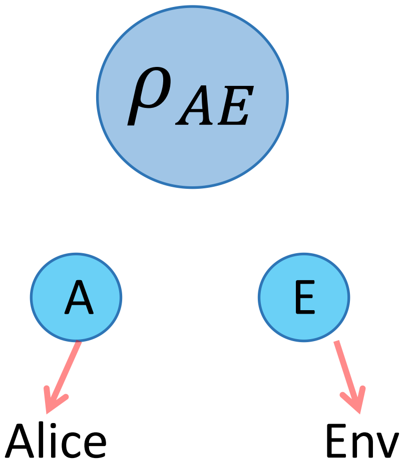
Outline

- Basic framework of quantum information decoupling
- Main result—one-shot bound on decoupling performance
- Applications in quantum information theory
  - Quantum state merging
  - Entanglement distillation
  - LOCC-assisted quantum channel coding
- Summaries

Quantum information decoupling—Definition



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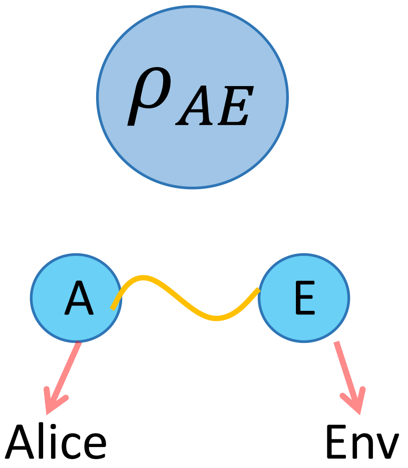

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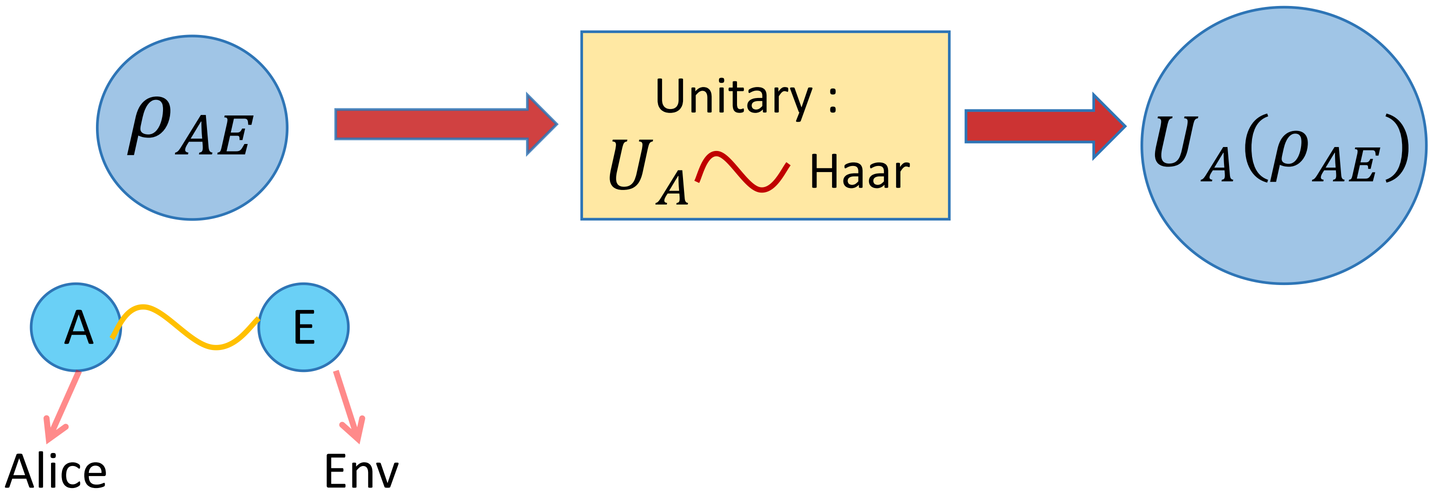

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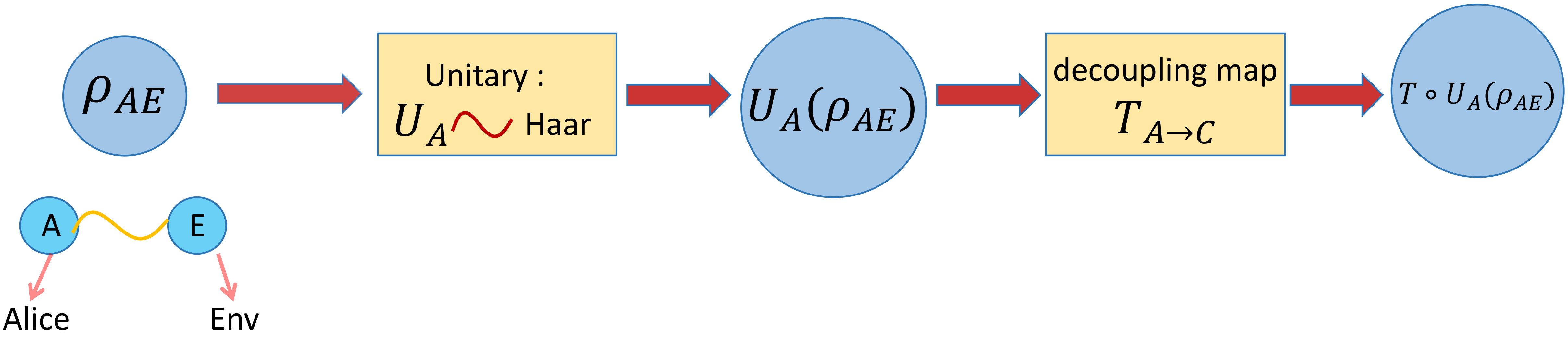

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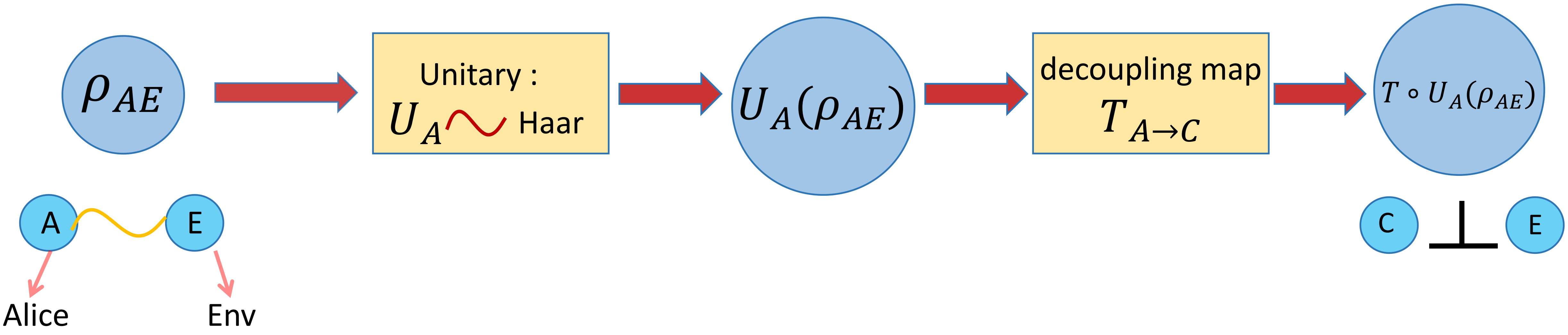
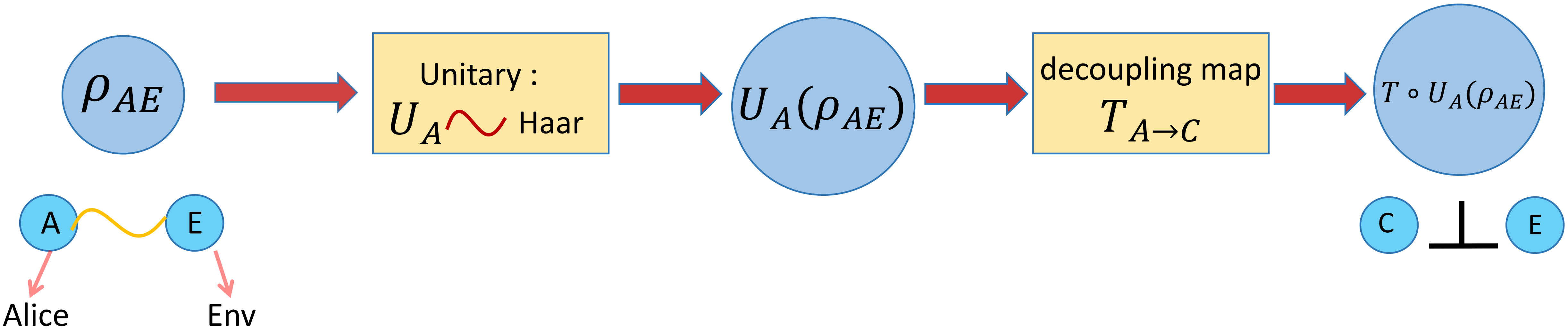
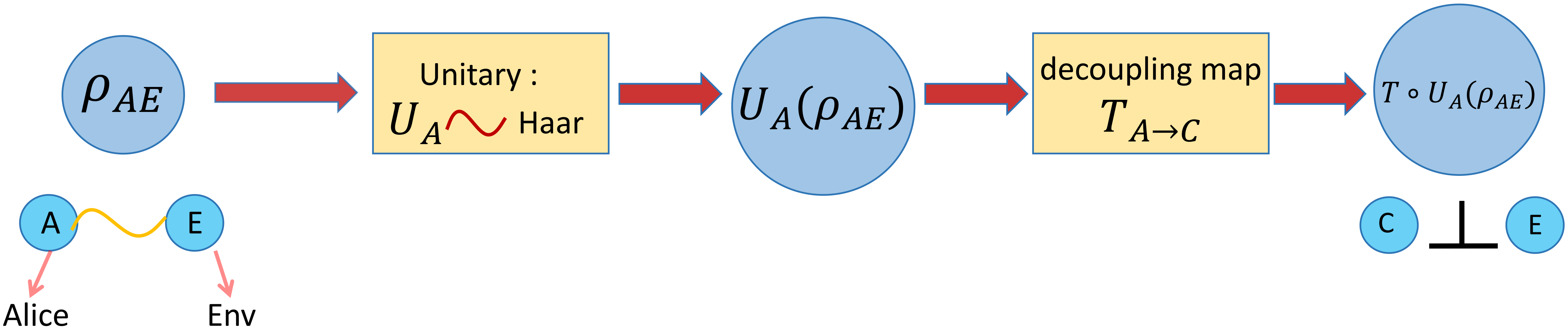


Quantum information decoupling—Definition



Question: How well can we decouple C from E ?

Quantum information decoupling—Definition



Question: How well can we decouple C from E ?

decoupling performance: $\mathbb{E}_{U_A\sim Haar}\, d(T\circ U_A(\rho_{AE}), \omega_C \otimes \rho_E), \quad \omega_C = T_{A\to C}(\frac{I_A}{|A|})$

Quantum information decoupling—Definition

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d

trace distance : $\Delta(\rho, \sigma) := \frac{1}{2} \| \rho - \sigma \|_1$

purified distance : $\mathrm{P}(\rho, \sigma) := \sqrt{1 - \| \sqrt{\rho}\sqrt{\sigma} \|_1^2}$

relative entropy: $\mathrm{D}(\rho \| \sigma) := \mathrm{Tr}\rho log \rho - \mathrm{Tr}\rho log \sigma$

Quantum information decoupling—Previous work

[Dupuis, Berta, Wullschleger, Renner: arxiv: 1012.6044]

Theorem 1 *For any bipartite state ρ_{AE}, decoupling map $T_{A\to C}$ and $\epsilon > 0$, we have*

$$E_{U_A\sim Haar}\frac{1}{2}\|T_{A\to C}\circ U_A(\rho_{AE}) - \omega_C\otimes\rho_E\|_1 \leq 2^{-\frac{1}{2}H^\epsilon_{\min}(A|E)_\rho - \frac{1}{2}H^\epsilon_{\min}(\bar{A}|C)_\omega} + 12\epsilon,$$

where $\omega_{\bar{A}C} := T_{A\to C}(\Phi_{\bar{A}A})$ is the Choi state of $T_{A\to C}$ and $H^\epsilon_{\min}(A|E)_\rho := -\inf_{\sigma_E} D^\epsilon_{\max}(\rho_{AE}\|I_A\otimes\sigma_E)$ is the smoothed conditional-min entropy.

Quantum information decoupling—Previous work

[Cheng, Dupuis, Gao: arxiv: 2409.15149]

Theorem 2 *For any bipartite state ρ_{AE}, decoupling map $T_{A\to C}$, we have*

$$E_{U_A\sim Haar}\frac{1}{2}\|T_{A\to C}\circ U_A(\rho_{AE})-\omega_C\otimes\rho_E\|_1\leq\inf_{1<\alpha<2}2^{\frac{1-\alpha}{\alpha}(H_\alpha^\uparrow(A|E)_\rho+H_\alpha^\uparrow(\bar{A}|C)_\omega+\log 3)},$$

where $H_\alpha^\uparrow(A|E)_\rho := -\inf_{\sigma_E} D_\alpha(\rho_{AE}\|I_A\otimes\sigma_E)$ *is the sandwiched Rényi conditional entropy.*

Quantum information decoupling—Previous work

	decoupling performance
trace distance	broadly studied
purified distance	unknown
relative entropy	unknown

Quantum information decoupling—Our work

Theorem 3 *For any bipartite state ρ_{AE}, decoupling map $T_{A\to C}$, we have*

$$E_{U_A\sim Haar} D(T_{A\to C}\circ U_A(\rho_{AE})\|\omega_C\otimes\rho_E) \leq \inf_{0<s<1} \frac{s^s(1-s)^{(1-s)}}{s} 2^{-s(H^{\downarrow}_{1+s}(A|E)_\rho + H^{\downarrow}_{1+s}(\bar{A}|C)_\omega)},$$

where $H^{\downarrow}_{1+s}(A|E)_\rho := -D_{1+s}(\rho_{AE}\|I_A\otimes\rho_E)$.

Quantum information decoupling—Our work

Proof sketch:

$$E_{U_A \sim Haar} D(T_{A \to C} \circ U_A(\rho_{AE}) \| \omega_C \otimes \rho_E)$$

Quantum information decoupling—Our work

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$$E_{U_A \sim Haar} D(T_{A\to C} \circ U_A(\rho_{AE}) \| \omega_C \otimes \rho_E)$$

$$= E_{U_A \sim Haar} \operatorname{Tr} T_{A\to C} \circ U_A(\rho_{AE}) \log T_{A\to C} \circ U_A(\rho_{AE}) - E_{U_A \sim Haar} \operatorname{Tr} T_{A\to C} \circ U_A(\rho_{AE}) \log \omega_C \otimes \rho_E$$

Quantum information decoupling—Our work

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$$\|$$

$$-\operatorname{Tr}\omega_C\otimes\rho_E \log\omega_C\otimes\rho_E$$

Proof sketch:

Choi representation : $T_{A\to C}(\rho_{AE}) = |A|^2 \langle \Phi_{A\bar{A}} | \rho_{AE} \otimes \omega_{\bar{A}C} | \Phi_{A\bar{A}} \rangle$

Quantum information decoupling—Our work

Proof sketch:

$$E_{U_A \sim Haar} \operatorname{Tr} T_{A\to C} \circ U_A(\rho_{AE}) \log T_{A\to C} \circ U_A(\rho_{AE})$$

Choi representation : $T_{A\to C}(\rho_{AE}) = |A|^2 \langle \Phi_{A\bar{A}} | \rho_{AE} \otimes \omega_{\bar{A}C} | \Phi_{A\bar{A}} \rangle$

$$= E_{U_A \sim Haar} |A|^2 \operatorname{Tr}(\rho_{AE} \otimes \omega_{\bar{A}C}) \left(\log |A|^2 \operatorname{Tr}_{A'\bar{A}} (U^*_{\bar{A}}(\Phi_{A\bar{A}}) \otimes U^*_{\bar{A}'}(\Phi_{A'\bar{A}}) \otimes I_{CE})(I_{A\bar{A}} \otimes \rho_{A'E} \otimes \omega_{\bar{A}C})\right)$$

Quantum information decoupling—Our work

Proof sketch:

$$E_{U_A\sim Haar}|A|^2\,\mathrm{Tr}\left(\rho_{AE}\otimes\omega_{\bar{A}C}\right)\left(\log|A|^2\,\mathrm{Tr}_{A'\tilde{A}}(U_{\tilde{A}}^*(\Phi_{A\bar{A}})\otimes U_{A'}^*(\Phi_{A'\tilde{A}})\otimes I_{CE})(I_{A\bar{A}}\otimes\rho_{A'E}\otimes\omega_{\tilde{A}C})\right)$$

Operator concavity of log

$$\leq|A|^2\,\mathrm{Tr}\left(\rho_{AE}\otimes\omega_{\bar{A}C}\right)\left(\log|A|^2\,\mathrm{Tr}_{A'\tilde{A}}(E_{U_A\sim Haar}U_{\tilde{A}}^*(\Phi_{A\bar{A}})\otimes U_{A'}^*(\Phi_{A'\tilde{A}})\otimes I_{CE})(I_{A\bar{A}}\otimes\rho_{A'E}\otimes\omega_{\tilde{A}C})\right)$$

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Werner twirling channel: $\mathcal{E}(X_{AA'}) := E_{U_A \sim Haar} U_A \otimes U_{A'}(X_{AA'})$

Quantum information decoupling—Our work

Proof sketch:

$$E_{U_A\sim Haar}|A|^2\operatorname{Tr}\left(\rho_{AE}\otimes\omega_{\bar{A}C}\right)\left(\log|A|^2\operatorname{Tr}_{A'\tilde{A}}(U_A^*(\Phi_{A\bar{A}})\otimes U_{A'}^*(\Phi_{A'\tilde{A}})\otimes I_{CE})(I_{A\bar{A}}\otimes\rho_{A'E}\otimes\omega_{\tilde{A}C})\right)$$

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Werner twirling channel: $\mathcal{E}(X_{AA'}):=E_{U_A\sim Haar}U_A\otimes U_{A'}(X_{AA'})$

$$=|A|^2\operatorname{Tr}\rho_{AE}\otimes\omega_{\bar{A}C}\log\Big(\frac{1}{|A|^2-1}I_{A\bar{A}}\otimes\rho_E\otimes\omega_C+\frac{1}{|A|^2-1}\rho_{AE}\otimes\omega_{\bar{A}C}-\frac{1}{|A|^3-|A|}\rho_{AE}\otimes I_{\bar{A}}\otimes\omega_C$$
$$-\frac{1}{|A|^3-|A|}I_A\otimes\rho_E\otimes\omega_{\bar{A}C}\Big)$$

Quantum information decoupling—Our work

Proof sketch:

$$E_{U_A\sim Haar}|A|^2\operatorname{Tr}(\rho_{AE}\otimes\omega_{\bar{A}C})\left(\log|A|^2\operatorname{Tr}_{A'\tilde{A}}(U_{\bar{A}}^*(\Phi_{A\bar{A}})\otimes U_{A'}^*(\Phi_{A'\tilde{A}})\otimes I_{CE})(I_{A\bar{A}}\otimes\rho_{A'E}\otimes\omega_{\tilde{A}C})\right)$$

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$$\leq\rho_{AE}\otimes\omega_{\bar{A}C}+I_{A\bar{A}}\otimes\rho_E\otimes\omega_C$$

Quantum information decoupling—Our work

Proof sketch:

$$E_{U_A \sim Haar} D(T_{A \to C} \circ U_A(\rho_{AE}) \| \omega_C \otimes \rho_E)$$

$$\leq \operatorname{Tr} \rho_{AE} \otimes \omega_{\bar{A}C} \log(\rho_{AE} \otimes \omega_{\bar{A}C} + I_{A\bar{A}} \otimes \rho_E \otimes \omega_C) - \operatorname{Tr} \rho_{AE} \otimes \omega_{\bar{A}C} \log I_{A\bar{A}} \otimes \rho_E \otimes \omega_C$$

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Trace inequality:

$$\operatorname{Tr} A \log(A+B) - \operatorname{Tr} A \log B \leq \inf_{0<s<1} \frac{s^s (1-s)^{(1-s)}}{s} 2^{s D_{1+s}(\rho\|\sigma)}, \ \forall A, B \geq 0$$

Quantum information decoupling—Our work

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$$\leq \inf_{0<s<1} \frac{s^s(1-s)^{(1-s)}}{s} 2^{-s(H^{\downarrow}_{1+s}(A|E)_\rho + H^{\downarrow}_{1+s}(\bar{A}|C)_\omega)}$$

Quantum information decoupling—Our work

i.i.d scenario:

$$E_{U_{A^n}\sim Haar} D(T_{A\to C}^{\otimes n} \circ U_{A^n}(\rho_{AE}^{\otimes n}) \| \omega_C^{\otimes n} \otimes \rho_E^{\otimes n}) \leq \inf_{0<s<1} \frac{s^s(1-s)^{(1-s)}}{s} 2^{-s(nH_{1+s}^{\downarrow}(A|E)_\rho + nH_{1+s}^{\downarrow}(\bar{A}|C)_\omega)}$$

Quantum information decoupling—Our work

i.i.d scenario:

$$E_{U_{A^n}\sim Haar} D(T_{A\to C}^{\otimes n}\circ U_{A^n}(\rho_{AE}^{\otimes n})\|\omega_C^{\otimes n}\otimes\rho_E^{\otimes n}) \leq \inf_{0<s<1} \frac{s^s(1-s)^{(1-s)}}{s} 2^{-s(nH_{1+s}^{\downarrow}(A|E)_\rho + nH_{1+s}^{\downarrow}(\bar{A}|C)_\omega)}$$

$\Downarrow$

$$\lim_{n\to\infty} \frac{-1}{n} \log E_{U_{A^n}\sim Haar} D(T_{A\to C}^{\otimes n}\circ U_{A^n}(\rho_{AE}^{\otimes n})\|\omega_C^{\otimes n}\otimes\rho_E^{\otimes n}) \geq \sup_{0<s<1} s(H_{1+s}^{\downarrow}(A|E)_\rho + H_{1+s}^{\downarrow}(\bar{A}|C)_\omega)$$

Quantum information decoupling—Our work

Theorem 4 *For any bipartite state ρ_{AE} and partial trace channel $T_{A\to C}$, we have*

$$\lim_{n\to\infty}\frac{-1}{n}\log E_{U_{A^n}\sim Haar} D(T_{A\to C}^{\otimes n}\circ U_{A^n}(\rho_{AE}^{\otimes n})\|\omega_C^{\otimes n}\otimes\rho_E^{\otimes n}) = \sup_{0<s<1} s(H_{1+s}^{\downarrow}(A|E)_\rho + H_{1+s}^{\downarrow}(\bar{A}|C)_\omega).$$

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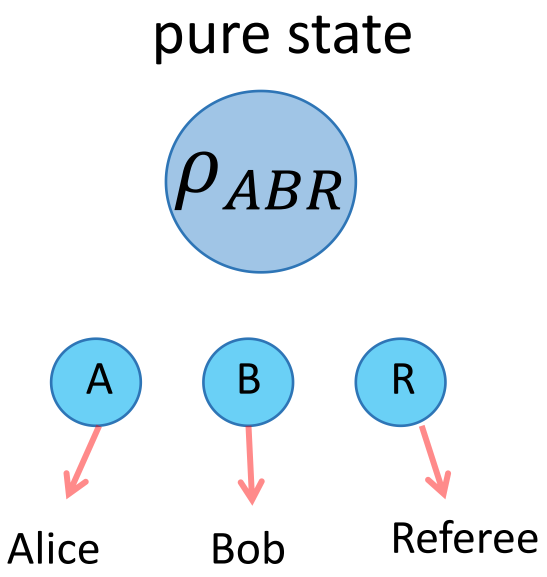
Open question:

$$\lim_{n\to\infty}\frac{-1}{n}\log E_{U_{A^n}\sim Haar} D(T_{A\to C}^{\otimes n}\circ U_{A^n}(\rho_{AE}^{\otimes n})\|\omega_C^{\otimes n}\otimes\rho_E^{\otimes n}) \overset{?}{=} \sup_{0<s<1} s(H_{1+s}^{\downarrow}(A|E)_\rho + H_{1+s}^{\downarrow}(\bar{A}|C)_\omega)$$

$,\forall$ *decoupling map* $T_{A\to C}$

Applications—Quantum state merging



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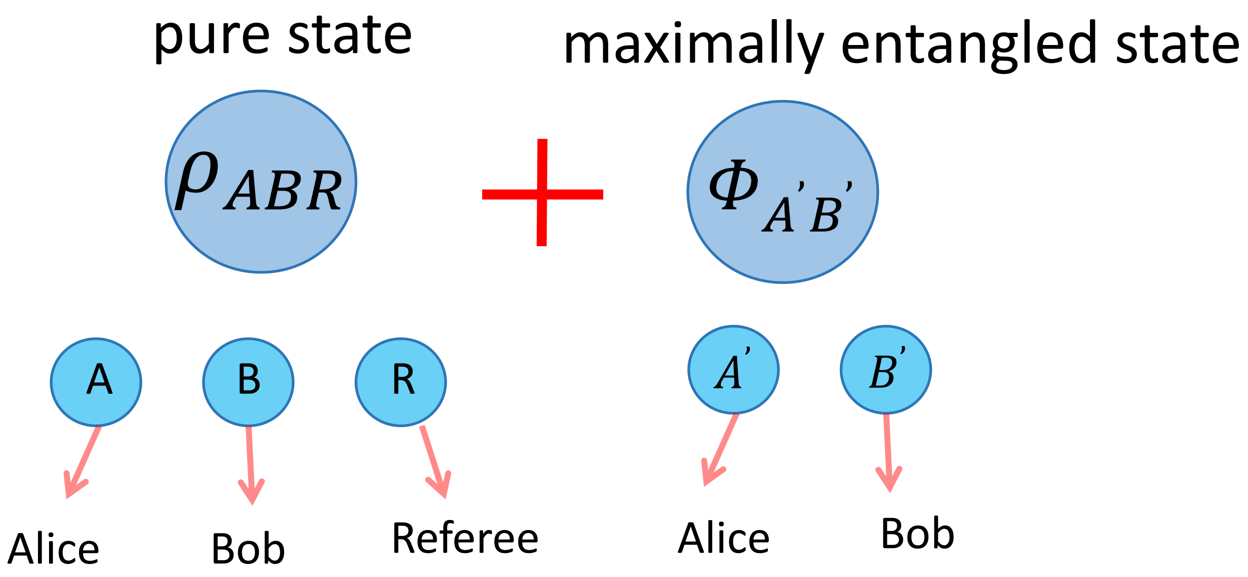

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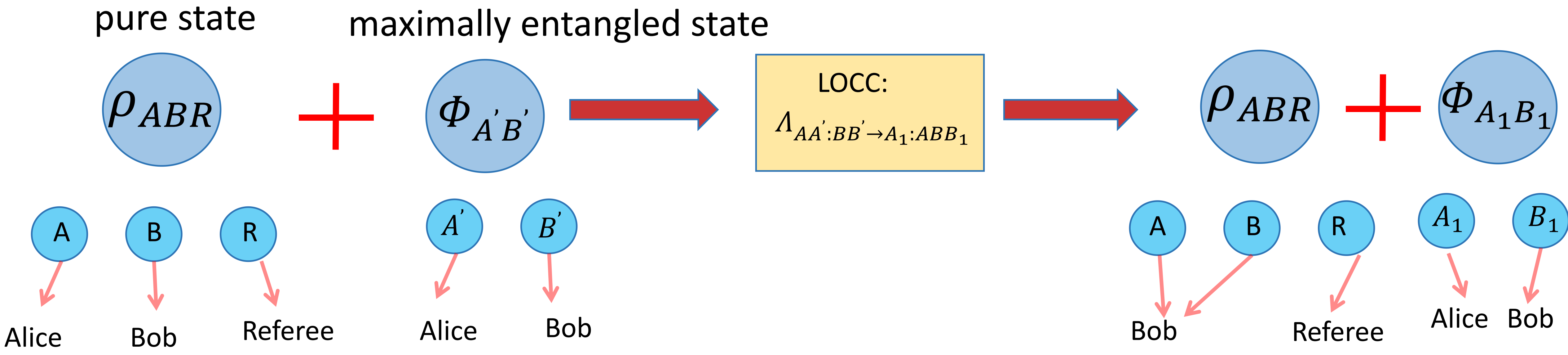

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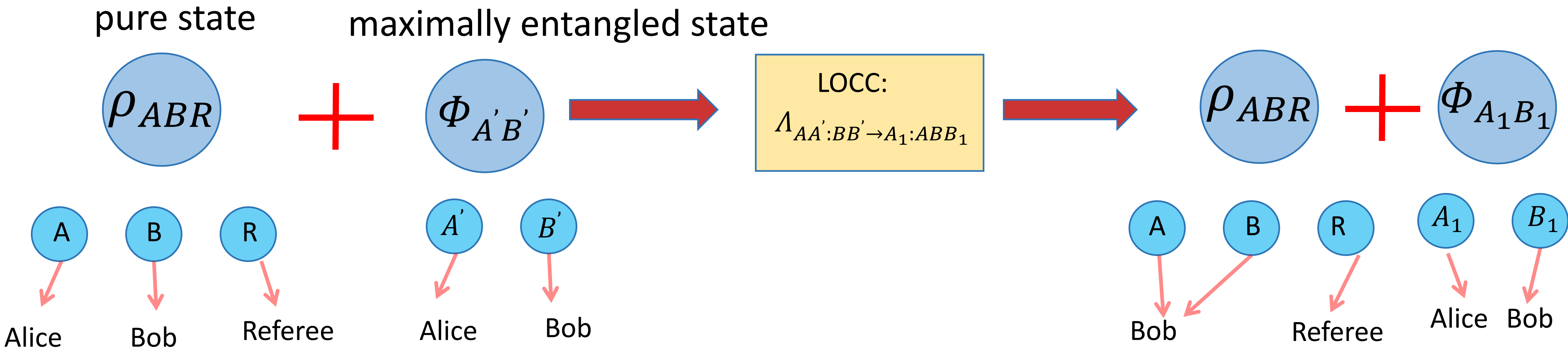

Performance: $P(\Lambda_{AA':BB' \to A_1:ABB_1}(\rho_{ABR} \otimes \Phi_{A'B'}), \rho_{ABR} \otimes \Phi_{A_1B_1})$

Quantum state merging protocol :

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Step 1: Decomposition of Hilbert space H_A

$$H_A = \bigoplus_{i=1}^{n} H_i, \quad |H_1| = |H_2| = \ldots = |H_m|$$

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$$H_A = \bigoplus_{i=1}^{n} H_i, \quad |H_1| = |H_2| = \ldots = |H_m|$$

Step 2: Construction of decoupling maps

partial isometry: $V_i^A : H_i \to H_1, \quad \forall i = 1, \ldots, m$

Quantum state merging protocol :

Step 3: Evaluation of performances of these decoupling maps

$$E_{U_A\sim Haar} D(V_i^A \circ U_A(\rho_{AR}) \| \frac{\Pi_{A_1}}{|A|} \otimes \rho_R) \leq \frac{s^s(1-s)^{(1-s)}}{s} \frac{|A_1|^{1+s}}{|A|} 2^{-sH_{1+s}(A|R)_\rho}, \quad \forall i = 1, \ldots, m$$

Quantum state merging protocol :

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$$\boxed{P(\rho, \sigma) \leq \sqrt{D(\rho\|\sigma)}}$$

Quantum state merging protocol :

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$$E_{U_A\sim Haar} D(V_i^A \circ U_A(\rho_{AR}) \| \frac{\Pi_{A_1}}{|A|} \otimes \rho_R) \leq \frac{s^s(1-s)^{(1-s)}}{s} \frac{|A_1|^{1+s}}{|A|} 2^{-sH_{1+s}(A|R)_\rho}, \quad \forall i = 1, \ldots, m$$

$$P(\rho, \sigma) \leq \sqrt{D(\rho\|\sigma)}$$

$$E_{U_A\sim Haar} \sum_{i=1}^{m} \operatorname{Tr} V_i^A \circ U_A(\rho_{AR}) P^2\left(\frac{V_i^A \circ U_A(\rho_{AR})}{\operatorname{Tr} V_i^A \circ U_A(\rho_{AR})}, \frac{\Pi_{A_1}}{|A_1|} \otimes \rho_R\right) \leq \frac{s^s(1-s)^{(1-s)}}{s} |A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

Quantum state merging protocol :

Step 4: Construction of the protocol via Uhlmann's theorem

$$E_{U_A\sim Haar}\sum_{i=1}^{m}\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})P^2\Big(\frac{V_i^A\circ U_A(\rho_{AR})}{\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})},\frac{\Pi_{A_1}}{|A_1|}\otimes\rho_R\Big)\leq\frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

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$$\downarrow$$

$$\frac{V_i^A\circ U_A(\rho_{ABR})}{\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})}$$

Quantum state merging protocol :

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↓ ↓

$$\frac{V_i^A\circ U_A(\rho_{ABR})}{\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})} \qquad \Phi_{A_1B_1}\otimes\rho_{ABR}$$

Quantum state merging protocol :

Step 4: Construction of the protocol via Uhlmann's theorem

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$$\|$$

$$P^2\left(\frac{V_i^A\circ U_A\otimes W_i^{B\to ABB_1}(\rho_{ABR})}{\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})},\Phi_{A_1B_1}\otimes\rho_{ABR}\right)$$

Quantum state merging protocol :

Step 4: Construction of the protocol via Uhlmann’s theorem

$$E_{U_A\sim Haar}\sum_{i=1}^{m}\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})P^2\left(\frac{V_i^A\circ U_A(\rho_{AR})}{\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})},\frac{\Pi_{A_1}}{|A_1|}\otimes\rho_R\right)\leq\frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

$$P^2\left(\frac{V_i^A\circ U_A\otimes W_i^{B\to ABB_1}(\rho_{ABR})}{\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})},\Phi_{A_1B_1}\otimes\rho_{ABR}\right)$$

$$E_{U_A\sim Haar}P^2\left(\sum_{i=1}^{m}V_i^A\circ U_A\otimes W_i^{B\to ABB_1}(\rho_{ABR}),\Phi_{A_1B_1}\otimes\rho_{ABR}\right)\leq\frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

Quantum state merging protocol :

Step 4: Construction of the protocol via Uhlmann's theorem

$$E_{U_A\sim Haar}\sum_{i=1}^{m}\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})P^2\Big(\frac{V_i^A\circ U_A(\rho_{AR})}{\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})},\frac{\Pi_{A_1}}{|A_1|}\otimes\rho_R\Big)\leq\frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

$$\Updownarrow$$

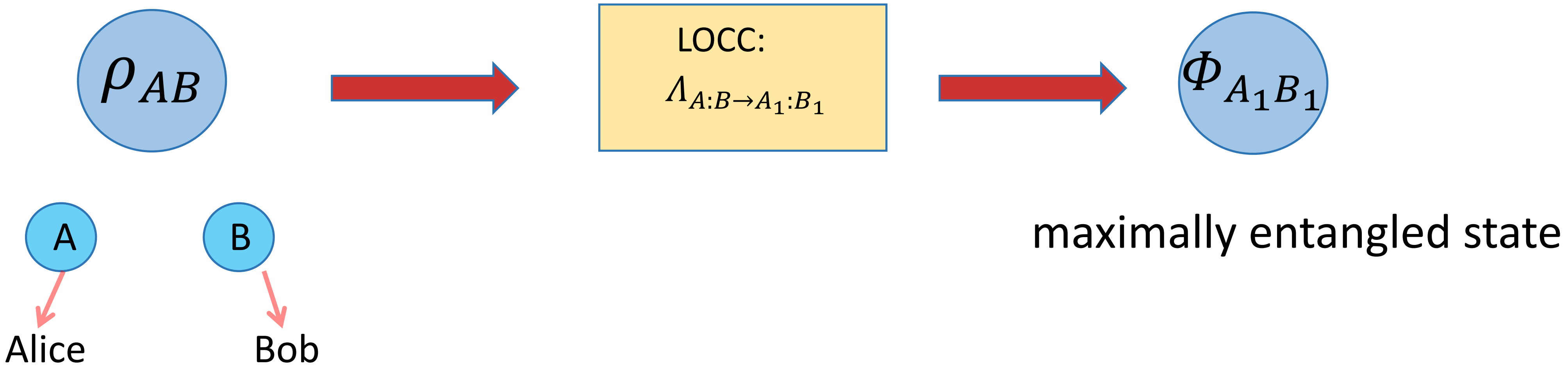
$$P^2\Big(\frac{V_i^A\circ U_A\otimes W_i^{B\to ABB_1}(\rho_{ABR})}{\operatorname{Tr} V_i^A\circ U_A(\rho_{AR})},\Phi_{A_1B_1}\otimes\rho_{ABR}\Big)$$

$$\Downarrow$$

$$E_{U_A\sim Haar}P^2\Big(\sum_{i=1}^{m}V_i^A\circ U_A\otimes W_i^{B\to ABB_1}(\rho_{ABR}),\Phi_{A_1B_1}\otimes\rho_{ABR}\Big)\leq\frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

LOCC protocol: $\sum_{i=1}^{m}V_i^A\circ U_A\otimes W_i^{B\to ABB_1}$

Applications—Entanglement distillation



Entanglement distillation protocol :

$$E_{U_A \sim Haar} P^2\left(\sum_{i=1}^{m} V_i^A \circ U_A \otimes W_i^{B \to ABB_1}(\rho_{ABR}), \Phi_{A_1B_1} \otimes \rho_{ABR}\right) \leq \frac{s^s(1-s)^{(1-s)}}{s} |A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

Entanglement distillation protocol :

$$E_{U_A \sim Haar} P^2\left(\sum_{i=1}^{m} V_i^A \circ U_A \otimes W_i^{B \to ABB_1}(\rho_{ABR}), \Phi_{A_1B_1} \otimes \rho_{ABR}\right) \leq \frac{s^s(1-s)^{(1-s)}}{s} |A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

↓

Trace out the system A, B and R

Entanglement distillation protocol :

$$E_{U_A\sim Haar}P^2(\sum_{i=1}^{m} V_i^A \circ U_A \otimes W_i^{B\to ABB_1}(\rho_{ABR}), \Phi_{A_1B_1}\otimes\rho_{ABR}) \le \frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

⇓ Trace out the system A, B and R

$$E_{U_A\sim Haar}P^2(\sum_{i=1}^{m} V_i^A \circ U_A \otimes W_i^{B\to B_1}(\rho_{AB}), \Phi_{A_1B_1}) \le \frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

Entanglement distillation protocol :

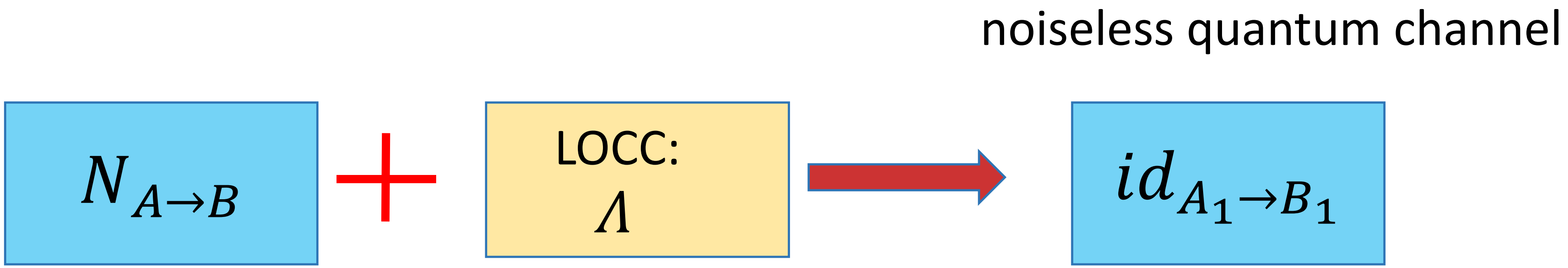
$$E_{U_A\sim Haar}P^2(\sum_{i=1}^{m} V_i^A \circ U_A \otimes W_i^{B\to ABB_1}(\rho_{ABR}), \Phi_{A_1B_1}\otimes\rho_{ABR}) \leq \frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

Trace out the system A, B and R

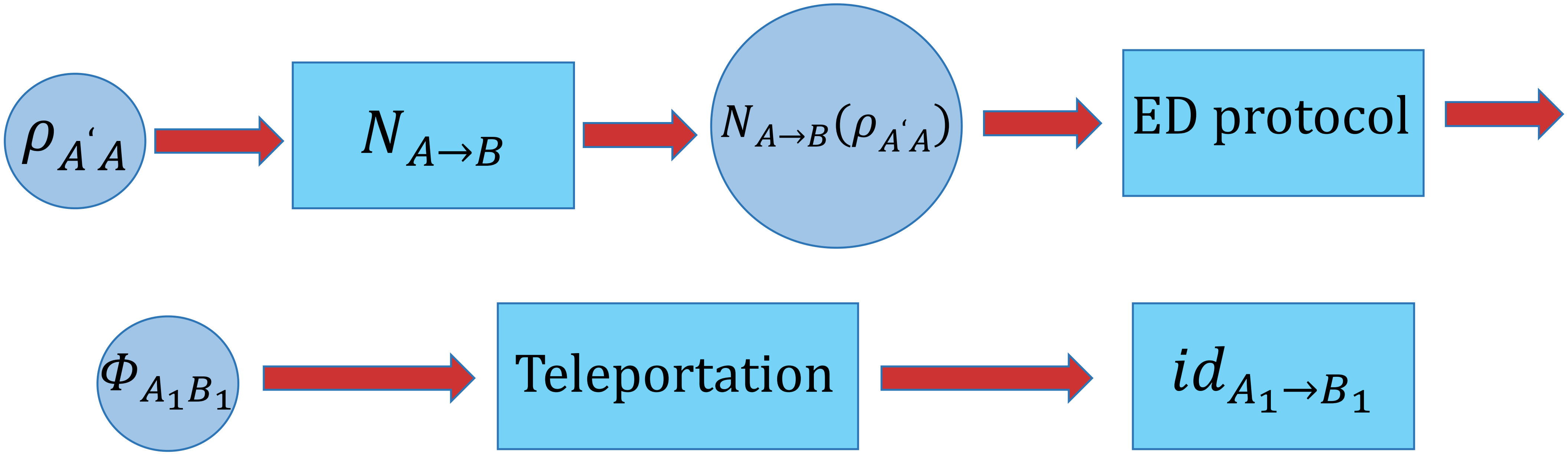
$$E_{U_A\sim Haar}P^2(\sum_{i=1}^{m} V_i^A \circ U_A \otimes W_i^{B\to B_1}(\rho_{AB}), \Phi_{A_1B_1}) \leq \frac{s^s(1-s)^{(1-s)}}{s}|A_1|^s 2^{-sH_{1+s}(A|R)_\rho}$$

Entanglement distillation protocol: $\sum_{i=1}^{m} V_i^A \circ U_A \otimes W_i^{B\to B_1}$

Applications—LOCC-assisted quantum channel coding



LOCC-assisted quantum channel coding protocol :



maximally entangled state

Applications—Error exponents for these tasks

	Achievability	Optimality
Quantum state merging	✓	✓
Entanglement distillation	✓	?
Locc-assisted channel coding	✓	?

Summary

- We establish a one-shot decoupling theorem under the relative entropy measure and derive an achievability bound on the error exponent for decoupling.
- Our main result can be used to establish one-shot upper bounds for quantum state merging, entanglement distillation and LOCC-assisted channel coding and achievable error exponents for these tasks.
- Our achievable error exponents are tight for some states and quantum channels. Therefore, the exact error exponents can be determined in these special cases.

Thank you!