

Computational Complexity of Optimally Encoding a Qubit

Beyond IID 14, Shenzhen, 2026

Idris Delsol¹, Omar Fawzi¹, Akshay Ramachandran²

June 25, 2026

¹ENS Lyon, Inria, ²University of British Columbia

The logo for Inria, featuring the word "Inria" in a stylized, red, cursive script font.The logo for ENS DE LYON, consisting of the letters "ENS" in a bold, black, sans-serif font with horizontal bars through them, and "ENS DE LYON" in a smaller, black, sans-serif font below it.

Motivation - Sending information

- ▶ Main bottleneck for application-scale Quantum Computers: **fragility** of quantum information [Terhal 2012]

Motivation - Sending information

- ▶ Main bottleneck for application-scale Quantum Computers: **fragility** of quantum information [Terhal 2012]
- ▶ **If the noise is weak enough: perfect transmission is asymptotically possible!**
(Existence of Quantum Error Correcting Codes [Shor 1996], first fault-tolerant computational scheme [Kitaev 1997])

Motivation - Sending information

- ▶ Main bottleneck for application-scale Quantum Computers: **fragility** of quantum information [Terhal 2012]
- ▶ **If the noise is weak enough: perfect transmission is asymptotically possible!**
(Existence of Quantum Error Correcting Codes [Shor 1996], first fault-tolerant computational scheme [Kitaev 1997])
- ▶ Established Quantum Error Correction as a major field

Motivation - Sending information

- ▶ Main bottleneck for application-scale Quantum Computers: **fragility** of quantum information [Terhal 2012]
- ▶ **If the noise is weak enough: perfect transmission is asymptotically possible!**
(Existence of Quantum Error Correcting Codes [Shor 1996], first fault-tolerant computational scheme [Kitaev 1997])
- ▶ Established Quantum Error Correction as a major field
- ▶ Maximum correctable 'noise level': **threshold**

Motivation - Sending information

- ▶ Main bottleneck for application-scale Quantum Computers: **fragility** of quantum information [Terhal 2012]
- ▶ **If the noise is weak enough: perfect transmission is asymptotically possible!** (Existence of Quantum Error Correcting Codes [Shor 1996], first fault-tolerant computational scheme [Kitaev 1997])
- ▶ Established Quantum Error Correction as a major field
- ▶ Maximum correctable 'noise level': **threshold**
- ▶ What is the **threshold** for the **depolarising channel** $\mathcal{D}_p(\rho) = (1 - p)\rho + p \operatorname{tr}(\rho) \frac{I}{d}$ with $\rho \in \mathbb{C}^{d \times d}$?

Motivation - Sending information

- ▶ Main bottleneck for application-scale Quantum Computers: **fragility** of quantum information [Terhal 2012]
- ▶ **If the noise is weak enough: perfect transmission is asymptotically possible!** (Existence of Quantum Error Correcting Codes [Shor 1996], first fault-tolerant computational scheme [Kitaev 1997])
- ▶ Established Quantum Error Correction as a major field
- ▶ Maximum correctable 'noise level': **threshold**
- ▶ What is the **threshold** for the **depolarising channel** $\mathcal{D}_p(\rho) = (1 - p)\rho + p \operatorname{tr}(\rho) \frac{I}{d}$ with $\rho \in \mathbb{C}^{d \times d}$?

Amounts to decide whether the **quantum capacity** is *strictly* positive

Motivation - Sending information

- ▶ Main bottleneck for application-scale Quantum Computers: **fragility** of quantum information [Terhal 2012]
- ▶ **If the noise is weak enough: perfect transmission is asymptotically possible!** (Existence of Quantum Error Correcting Codes [Shor 1996], first fault-tolerant computational scheme [Kitaev 1997])
- ▶ Established Quantum Error Correction as a major field
- ▶ Maximum correctable 'noise level': **threshold**
- ▶ What is the **threshold** for the **depolarising channel** $\mathcal{D}_p(\rho) = (1 - p)\rho + p \text{tr}(\rho) \frac{I}{d}$ with $\rho \in \mathbb{C}^{d \times d}$?

Amounts to decide whether the **quantum capacity** is *strictly* positive

Unknown for general channels

Depolarising channel - Best bounds on the quantum capacity

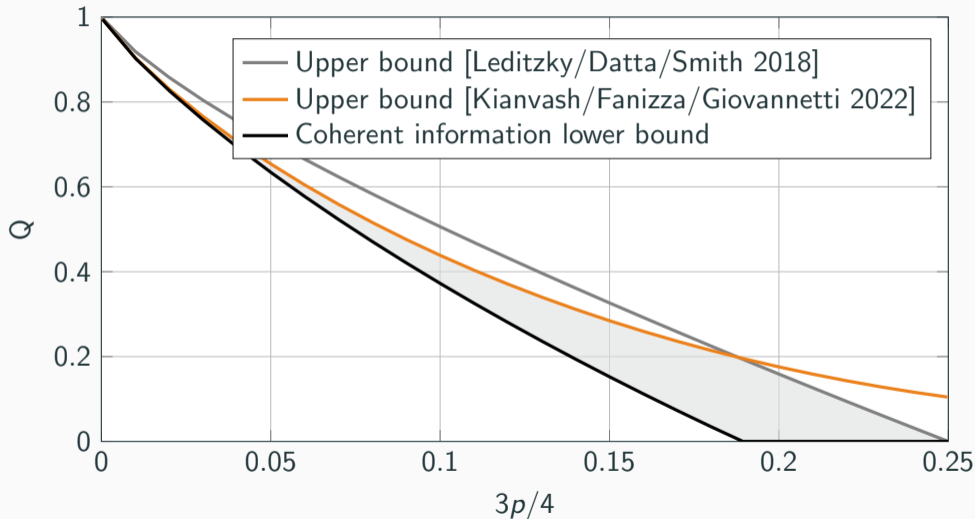


Figure 1: Best known bounds for the depolarising channel capacity

Simpler but related problem: **channel coding**

Channel Coding

Simpler but related problem: channel coding

Given a noise channel \mathcal{N} , how to best encode information?

Simpler but related problem: channel coding

Given a noise channel \mathcal{N} , how to best encode information?

Input A channel \mathcal{N} .

Messages

- ▶ Classical: $i \in [k] = \{1, \dots, k\}$
- ▶ Quantum: $|\psi\rangle \in \mathbb{C}^d$ (topic of this talk).

Goal Find an encoder and a decoder that maximises transmission fidelity.

Simpler but related problem: channel coding

Given a noise channel \mathcal{N} , how to best encode information?

Input A channel \mathcal{N} .

- Messages**
- ▶ Classical: $i \in [k] = \{1, \dots, k\}$
 - ▶ Quantum: $|\psi\rangle \in \mathbb{C}^d$ (topic of this talk).

Goal Find an encoder and a decoder that maximises transmission fidelity.

- ▶ If we can achieve perfect transmission fidelity, the capacity is strictly positive

Simpler but related problem: channel coding

Given a noise channel \mathcal{N} , how to best encode information?

Input A channel \mathcal{N} .

- Messages**
- ▶ Classical: $i \in [k] = \{1, \dots, k\}$
 - ▶ Quantum: $|\psi\rangle \in \mathbb{C}^d$ (topic of this talk).

Goal Find an encoder and a decoder that maximises transmission fidelity.

- ▶ If we can achieve perfect transmission fidelity, the capacity is strictly positive
- ▶ If the capacity is strictly positive, the optimal encoding to $\mathcal{N}^{\otimes k}$ tends to perfect transmission as $k \rightarrow \infty$

Channel Coding

Simpler but related problem: **channel coding**

Given a noise channel \mathcal{N} , how to **best encode** information?

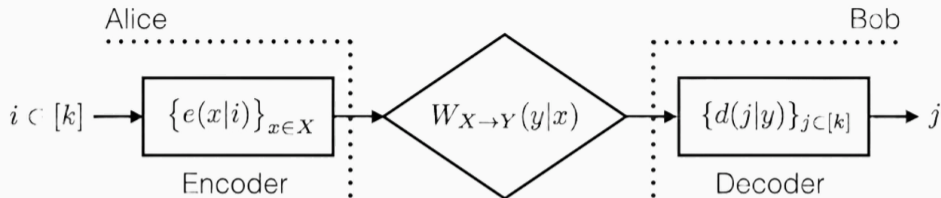
Input A channel \mathcal{N} .

- Messages**
- ▶ Classical: $i \in [k] = \{1, \dots, k\}$
 - ▶ Quantum: $|\psi\rangle \in \mathbb{C}^d$ (**topic of this talk**).

Goal Find an encoder and a decoder that maximises transmission fidelity.

- ▶ If we can achieve **perfect transmission** fidelity, the **capacity** is **strictly positive**
- ▶ If the **capacity** is **strictly positive**, the optimal encoding to $\mathcal{N}^{\otimes k}$ tends to **perfect transmission** as $k \rightarrow \infty$
- ▶ If \mathcal{N} given by a quantum circuit, known to be QMA-hard [Beigi/Shor 2007], [Bhattacharyya/Mehta/Zhao 2026]

Classical Case



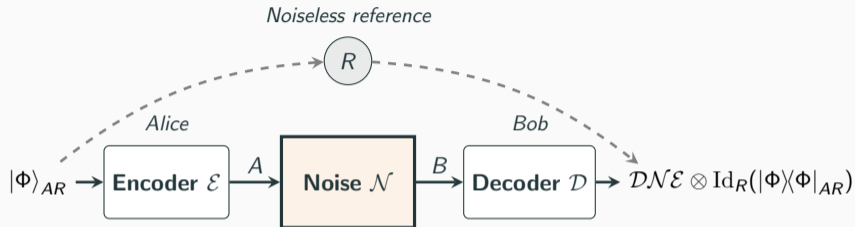
Objective: Compute the optimal encoding $p_{\text{succ}}(W, k) := \max_{e,d} \mathbf{P}_{i \sim [k]}[i = j]$.

Theorem [Barman/Fawzi 2018]

- ▶ Efficient algorithm to approximate $p_{\text{succ}}(W, k)$ up to a factor $(1 - e^{-1})$.
- ▶ Doing better is NP-hard.

Idea: classical channel coding \approx maximum coverage.

Entanglement Fidelity



- ▶ Relevant metric: how much does the channel preserve entanglement with the reference
- ▶ Maximally entangled state over 2 registers $|\Phi\rangle_{AR} = \frac{1}{\sqrt{k}}(\sum_{i=1}^k |ii\rangle)$
 - ▶ Measuring one register determines the other
 - ▶ The measure of a single register is uniform
- ▶ Given noise \mathcal{N} , encoder \mathcal{E} , decoder \mathcal{D} ,
$$F_e(\mathcal{D}\mathcal{N}\mathcal{E}) = \langle\Phi|_{BR} (\mathcal{D}\mathcal{N}\mathcal{E} \otimes \text{Id}_R)(|\Phi\rangle\langle\Phi|_{AR}) |\Phi\rangle_{BR}$$

Restrict to encoding a **qubit**.

Restrict to encoding a **qubit**.

Theorem

Given a classical description of a noise channel \mathcal{N} , deciding whether $\max_{\mathcal{D}, \mathcal{E}} F_e(\mathcal{D}\mathcal{N}\mathcal{E}) = 1$ with 2-dimensional input space is **NP-complete**.

Restrict to encoding a **qubit**.

Theorem

Given a classical description of a noise channel \mathcal{N} , deciding whether $\max_{\mathcal{D}, \mathcal{E}} F_e(\mathcal{D}\mathcal{N}\mathcal{E}) = 1$ with 2-dimensional input space is **NP-complete**.

Remark

- ▶ Easy algorithm for a classical channel with classical bit: decide whether its confusability graph has an **independent set** of size at least 2.
- ▶ Shows that deciding whether a **quantum** confusability graph has an independent set of size at least 2 is **NP-complete**

Restrict to encoding a **qubit**.

Theorem

Given a classical description of a noise channel \mathcal{N} , deciding whether $\max_{\mathcal{D}, \mathcal{E}} F_e(\mathcal{D}\mathcal{N}\mathcal{E}) = 1$ with 2-dimensional input space is **NP-complete**.

Remark

- ▶ Easy algorithm for a classical channel with classical bit: decide whether its confusability graph has an **independent set** of size at least 2.
- ▶ Shows that deciding whether a **quantum** confusability graph has an independent set of size at least 2 is **NP-complete**
- ▶ Also deciding whether a **quantum** channel can perfectly transmit a **classical bit** is **NP-complete** [D./Fawzi/Kochanowski/Ramachandran 2025]

Quantum Confusibility Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]

Quantum Confusability Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]
- ▶ Quantum confusability graph of Φ :

$$G_\Phi = \text{Span}\{K_i^* K_j \mid i, j \in [k]\}.$$

Quantum Confusability Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]
- ▶ Quantum confusability graph of Φ :

$$G_\Phi = \text{Span}\{K_i^* K_j \mid i, j \in [k]\}.$$

- ▶ If Φ classical

Quantum Confusability Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]
- ▶ Quantum confusability graph of Φ :

$$G_\Phi = \text{Span}\{K_i^* K_j \mid i, j \in [k]\}.$$

- ▶ If Φ classical
 - ▶ $\Phi(|i\rangle) = |\psi_i\rangle$, so $\Phi(x) = \sum_{i=1}^k |\psi_i\rangle\langle i| x |i\rangle\langle \psi_i|$

Quantum Confusability Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]
- ▶ Quantum confusability graph of Φ :

$$G_\Phi = \text{Span}\{K_i^* K_j \mid i, j \in [k]\}.$$

- ▶ If Φ classical
 - ▶ $\Phi(|i\rangle) = |\psi_i\rangle$, so $\Phi(x) = \sum_{i=1}^k |\psi_i\rangle\langle i| x |i\rangle\langle\psi_i|$
 - ▶ $G_\Phi = \text{Span}\{\langle\psi_i|\psi_j\rangle |i\rangle\langle j| \mid i, j \in [k]\}$

Quantum Confusability Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]
- ▶ Quantum confusability graph of Φ :

$$G_\Phi = \text{Span}\{K_i^* K_j \mid i, j \in [k]\}.$$

- ▶ If Φ classical
 - ▶ $\Phi(|i\rangle) = |\psi_i\rangle$, so $\Phi(x) = \sum_{i=1}^k |\psi_i\rangle\langle i| x |i\rangle\langle\psi_i|$
 - ▶ $G_\Phi = \text{Span}\{\langle\psi_i|\psi_j\rangle |i\rangle\langle j| \mid i, j \in [k]\}$
 - ▶ $|i\rangle\langle j| \in G_\Phi \iff \langle\psi_i|\psi_j\rangle \neq 0 \iff |\psi_i\rangle, |\psi_j\rangle$ physically confusable

Quantum Confusability Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]
- ▶ Quantum confusability graph of Φ :

$$G_\Phi = \text{Span}\{K_i^* K_j \mid i, j \in [k]\}.$$

- ▶ If Φ classical
 - ▶ $\Phi(|i\rangle) = |\psi_i\rangle$, so $\Phi(x) = \sum_{i=1}^k |\psi_i\rangle\langle i| x |i\rangle\langle\psi_i|$
 - ▶ $G_\Phi = \text{Span}\{\langle\psi_i|\psi_j\rangle |i\rangle\langle j| \mid i, j \in [k]\}$
 - ▶ $|i\rangle\langle j| \in G_\Phi \iff \langle\psi_i|\psi_j\rangle \neq 0 \iff |\psi_i\rangle, |\psi_j\rangle$ physically confusable
- ▶ $G_{\mathcal{D}_p} = \text{Span}\{I, X, Y, Z\} = \mathbb{C}^{2 \times 2}$ for $p > 0$

Quantum Confusability Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]
- ▶ Quantum confusability graph of Φ :

$$G_\Phi = \text{Span}\{K_i^* K_j \mid i, j \in [k]\}.$$

- ▶ If Φ classical
 - ▶ $\Phi(|i\rangle) = |\psi_i\rangle$, so $\Phi(x) = \sum_{i=1}^k |\psi_i\rangle\langle i| x |i\rangle\langle\psi_i|$
 - ▶ $G_\Phi = \text{Span}\{\langle\psi_i|\psi_j\rangle |i\rangle\langle j| \mid i, j \in [k]\}$
 - ▶ $|i\rangle\langle j| \in G_\Phi \iff \langle\psi_i|\psi_j\rangle \neq 0 \iff |\psi_i\rangle, |\psi_j\rangle$ physically confusable
- ▶ $G_{\mathcal{D}_p} = \text{Span}\{I, X, Y, Z\} = \mathbb{C}^{2 \times 2}$ for $p > 0$
- ▶ **Quantum independence number** of G_Φ : maximum rank of P orthogonal projector s.t. $P G_\Phi P = \mathbb{C}P$.

Quantum Confusability Graph

- ▶ If Φ is a quantum channel: $\Phi : x \mapsto \sum_{i=1}^k K_i x K_i^*$ [Kraus 1983]
- ▶ Quantum confusability graph of Φ :

$$G_\Phi = \text{Span}\{K_i^* K_j \mid i, j \in [k]\}.$$

- ▶ If Φ classical
 - ▶ $\Phi(|i\rangle) = |\psi_i\rangle$, so $\Phi(x) = \sum_{i=1}^k |\psi_i\rangle\langle i| x |i\rangle\langle\psi_i|$
 - ▶ $G_\Phi = \text{Span}\{\langle\psi_i|\psi_j\rangle |i\rangle\langle j| \mid i, j \in [k]\}$
 - ▶ $|i\rangle\langle j| \in G_\Phi \iff \langle\psi_i|\psi_j\rangle \neq 0 \iff |\psi_i\rangle, |\psi_j\rangle$ physically confusable
- ▶ $G_{\mathcal{D}_p} = \text{Span}\{I, X, Y, Z\} = \mathbb{C}^{2 \times 2}$ for $p > 0$
- ▶ **Quantum independence number** of G_Φ : maximum rank of P orthogonal projector s.t. $P G_\Phi P = \mathbb{C}P$.
- ▶ Perfect transmission through Φ iff $P G_\Phi P = \mathbb{C}P$ with $\text{rk}(P) \geq 2$ [Knill/Laflamme 1997].

Reduction sketch

Reduction sketch:

- ▶ Reduction from Graph 2-CSP. Let G be an instance of size n either satisfiable or 'far' from satisfiable
- ▶ NP-hard problem for some adequate parameter

Reduction sketch

Reduction sketch:

- ▶ Reduction from Graph 2-CSP. Let G be an instance of size n either satisfiable or 'far' from satisfiable
- ▶ NP-hard problem for some adequate parameter
- ▶ From G , by [Le Gall/Nakagawa/Nishimura 2012] (see also [Blier/Tapp 2009]), construct $A \in \mathcal{L}(\mathbb{C}^n \otimes \mathbb{C}^n)$ such that
 - ▶ G satisfiable $\implies \exists u \in \mathbb{C}^n, uu^* \otimes uu^* \in \ker A$
 - ▶ G 'far' from satisfiable $\implies \ell_{\text{SEP}}(A)$ 'far' from 0, with $\ell_{\text{SEP}}(A) = \min_{\omega \in \text{SEP}} \text{tr}(\omega A)$

Reduction sketch

Reduction sketch:

- ▶ Reduction from Graph 2-CSP. Let G be an instance of size n either satisfiable or 'far' from satisfiable
- ▶ NP-hard problem for some adequate parameter
- ▶ From G , by [Le Gall/Nakagawa/Nishimura 2012] (see also [Blier/Tapp 2009]), construct $A \in \mathcal{L}(\mathbb{C}^n \otimes \mathbb{C}^n)$ such that
 - ▶ G satisfiable $\implies \exists u \in \mathbb{C}^n, uu^* \otimes uu^* \in \ker A$
 - ▶ G 'far' from satisfiable $\implies \ell_{\text{SEP}}(A)$ 'far' from 0, with $\ell_{\text{SEP}}(A) = \min_{\omega \in \text{SEP}} \text{tr}(\omega A)$
 - ▶ Note: computing $\ell_{\text{SEP}}(M)$ is **NP-hard** for a **general** operator M , but A has **too much structure** to apply this result directly

Reduction sketch

Reduction sketch:

- ▶ Reduction from Graph 2-CSP. Let G be an instance of size n either satisfiable or 'far' from satisfiable
- ▶ NP-hard problem for some adequate parameter
- ▶ From G , by [Le Gall/Nakagawa/Nishimura 2012] (see also [Blier/Tapp 2009]), construct $A \in \mathcal{L}(\mathbb{C}^n \otimes \mathbb{C}^n)$ such that
 - ▶ G satisfiable $\implies \exists u \in \mathbb{C}^n, uu^* \otimes uu^* \in \ker A$
 - ▶ G 'far' from satisfiable $\implies \ell_{\text{SEP}}(A)$ 'far' from 0, with $\ell_{\text{SEP}}(A) = \min_{\omega \in \text{SEP}} \text{tr}(\omega A)$
 - ▶ Note: computing $\ell_{\text{SEP}}(M)$ is **NP-hard** for a **general** operator M , but A has **too much structure** to apply this result directly
- ▶ Let $\mathcal{S} = \text{vec}^{-1}(\ker(A))$, with $\text{vec}(|i\rangle\langle j|) = |ij\rangle$

Reduction sketch

Reduction sketch:

- ▶ Reduction from Graph 2-CSP. Let G be an instance of size n either satisfiable or 'far' from satisfiable
- ▶ NP-hard problem for some adequate parameter
- ▶ From G , by [Le Gall/Nakagawa/Nishimura 2012] (see also [Blier/Tapp 2009]), construct $A \in \mathcal{L}(\mathbb{C}^n \otimes \mathbb{C}^n)$ such that
 - ▶ G satisfiable $\implies \exists u \in \mathbb{C}^n, uu^* \otimes uu^* \in \ker A$
 - ▶ G 'far' from satisfiable $\implies \ell_{\text{SEP}}(A)$ 'far' from 0, with $\ell_{\text{SEP}}(A) = \min_{\omega \in \text{SEP}} \text{tr}(\omega A)$
 - ▶ Note: computing $\ell_{\text{SEP}}(M)$ is **NP-hard** for a **general** operator M , but A has **too much structure** to apply this result directly
- ▶ Let $\mathcal{S} = \text{vec}^{-1}(\ker(A))$, with $\text{vec}(|i\rangle\langle j|) = |ij\rangle$
- ▶ Let $\hat{\mathcal{S}} = \begin{pmatrix} \mathcal{S} + \mathcal{S}^* & \mathcal{S} \\ \mathcal{S}^* & -(\mathcal{S} + \mathcal{S}^*) \end{pmatrix}$. $\hat{\mathcal{S}}^\perp$ is a quantum confusability graph (or operator system)

Reduction sketch

Reduction sketch:

- ▶ Reduction from Graph 2-CSP. Let G be an instance of size n either satisfiable or 'far' from satisfiable
- ▶ NP-hard problem for some adequate parameter
- ▶ From G , by [Le Gall/Nakagawa/Nishimura 2012] (see also [Blier/Tapp 2009]), construct $A \in \mathcal{L}(\mathbb{C}^n \otimes \mathbb{C}^n)$ such that
 - ▶ G satisfiable $\implies \exists u \in \mathbb{C}^n, uu^* \otimes uu^* \in \ker A$
 - ▶ G 'far' from satisfiable $\implies \ell_{\text{SEP}}(A)$ 'far' from 0, with $\ell_{\text{SEP}}(A) = \min_{\omega \in \text{SEP}} \text{tr}(\omega A)$
 - ▶ Note: computing $\ell_{\text{SEP}}(M)$ is **NP-hard** for a **general** operator M , but A has **too much structure** to apply this result directly
- ▶ Let $\mathcal{S} = \text{vec}^{-1}(\ker(A))$, with $\text{vec}(|i\rangle\langle j|) = |ij\rangle$
- ▶ Let $\widehat{\mathcal{S}} = \begin{pmatrix} \mathcal{S} + \mathcal{S}^* & \mathcal{S} \\ \mathcal{S}^* & -(\mathcal{S} + \mathcal{S}^*) \end{pmatrix}$. $\widehat{\mathcal{S}}^\perp$ is a quantum confusability graph (or operator system)
- ▶ Construct \mathcal{N} such that $G_{\mathcal{N}} = \widehat{\mathcal{S}}^\perp$ [Duan 2009]

Reduction sketch

Reduction sketch:

- ▶ Reduction from Graph 2-CSP. Let G be an instance of size n either satisfiable or 'far' from satisfiable
- ▶ NP-hard problem for some adequate parameter
- ▶ From G , by [Le Gall/Nakagawa/Nishimura 2012] (see also [Blier/Tapp 2009]), construct $A \in \mathcal{L}(\mathbb{C}^n \otimes \mathbb{C}^n)$ such that
 - ▶ G satisfiable $\implies \exists u \in \mathbb{C}^n, uu^* \otimes uu^* \in \ker A$
 - ▶ G 'far' from satisfiable $\implies \ell_{\text{SEP}}(A)$ 'far' from 0, with $\ell_{\text{SEP}}(A) = \min_{\omega \in \text{SEP}} \text{tr}(\omega A)$
 - ▶ Note: computing $\ell_{\text{SEP}}(M)$ is **NP-hard** for a **general** operator M , but A has **too much structure** to apply this result directly
- ▶ Let $\mathcal{S} = \text{vec}^{-1}(\ker(A))$, with $\text{vec}(|i\rangle\langle j|) = |ij\rangle$
- ▶ Let $\hat{\mathcal{S}} = \begin{pmatrix} \mathcal{S} + \mathcal{S}^* & \mathcal{S} \\ \mathcal{S}^* & -(\mathcal{S} + \mathcal{S}^*) \end{pmatrix}$. $\hat{\mathcal{S}}^\perp$ is a quantum confusability graph (or operator system)
- ▶ Construct \mathcal{N} such that $G_{\mathcal{N}} = \hat{\mathcal{S}}^\perp$ [Duan 2009]
- ▶ G satisfiable \iff quantum independence number of $\hat{\mathcal{S}}^\perp = G_{\mathcal{N}}$ is $\geq 2 \iff \max_{\mathcal{D}, \mathcal{E}} F_e(\mathcal{DN}\mathcal{E}) = 1$

Summary and research direction

- ▶ For \mathcal{N} a noise channel, deciding if it can perfectly transmit information is NP-complete
- ▶ Given $k \in \mathbb{N}$, deciding whether k copies of \mathcal{N} (i.e. $\mathcal{N}^{\otimes k}$) can perfectly transmit information is NP-hard
- ▶ Deciding if a quantum graph is a quantum clique is coNP-complete
- ▶ Strong evidence that deciding whether the quantum capacity of \mathcal{N} is strictly positive is NP-hard

Summary and research direction

- ▶ For \mathcal{N} a noise channel, deciding if it can perfectly transmit information is NP-complete
- ▶ Given $k \in \mathbb{N}$, deciding whether k copies of \mathcal{N} (i.e. $\mathcal{N}^{\otimes k}$) can perfectly transmit information is NP-hard
- ▶ Deciding if a quantum graph is a quantum clique is coNP-complete
- ▶ Strong evidence that deciding whether the quantum capacity of \mathcal{N} is strictly positive is NP-hard

What's the picture for the **depolarising channel**?

Open problem - Asymptotic entanglement fidelity of the depolarising channel

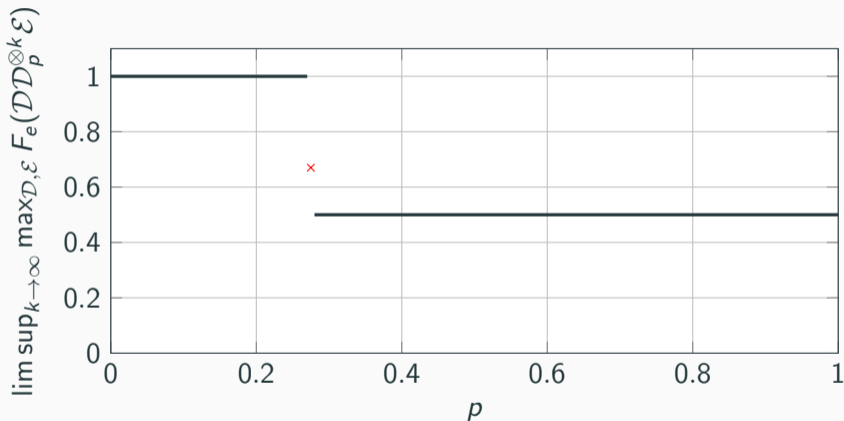


Figure 2: Conjecture for $\limsup_{k \rightarrow \infty} \max_{\mathcal{D}, \mathcal{E}} F_e(\mathcal{D}\mathcal{D}^{\otimes k}\mathcal{E})$