Fundamental Quality Bound on Optical Quantum Communication

based on arXiv:2510.07121

Tobias Rippchen, Ludovico Lami, Gerardo Adesso, Mario Berta

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Optical Quantum Communication

Alice and Bob are connected by noisy optical channel $\mathcal{N}_{A\to B}$
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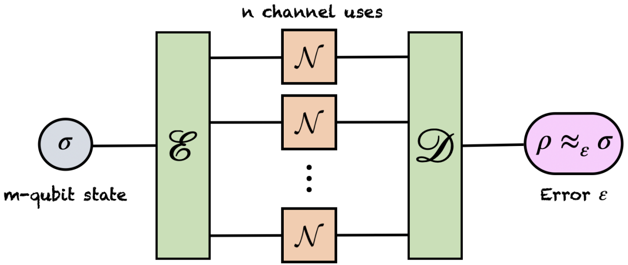
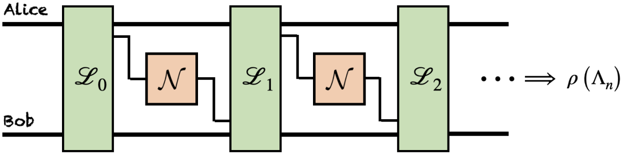
Quantum Communication:
Using the channel n times, transmit m qubits, with error ε.
Moreover, any correlations with reference system should be preserved!



Two-Way LOCC Assistance

Assistance Model: Adaptive two-way LOCC operations $\Lambda_n \in \mathrm{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n})$



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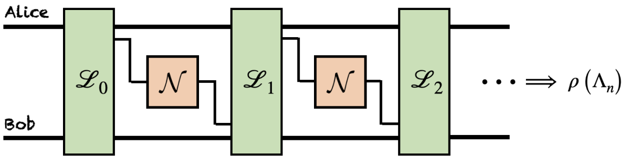


$\hookrightarrow$ Most-important practical assistance model

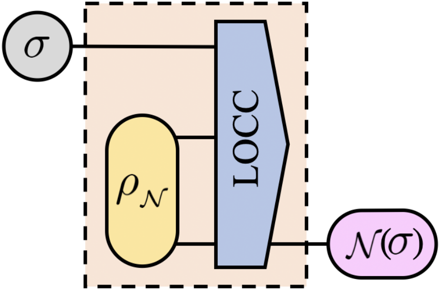
By quantum teleportation primitive: equivalent to entanglement distribution

Entanglement Distribution:
Find two-way assisted adaptive protocol $\Lambda_n \in \mathrm{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n})$ with output:

$$\rho(\Lambda_n) \approx_\varepsilon \Phi^{\otimes m} \quad \text{with } |\Phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Teleportation-Simulability

Most optical channels (e.g. thermal attenuator and amplifier, additive-Gaussian noise) are *teleportation-simulable* (with Choi state $\rho_{\mathcal{N}} := (\mathcal{N} \otimes \mathcal{I})(\Phi)$):



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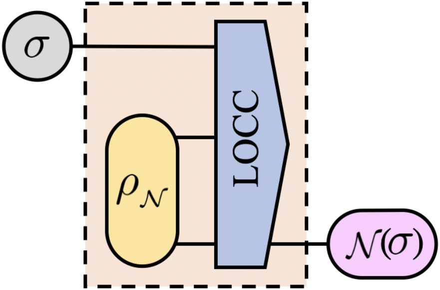

Adaptive-to-Block Reduction:[1]

For any $\Lambda_n \in \mathrm{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n})$: $\rho(\Lambda_n) = \bar{\Lambda}\left(\rho_{\mathcal{N}}^{\otimes n}\right)$ with $\bar{\Lambda} \in \mathrm{LOCC}_{\leftrightarrow}$

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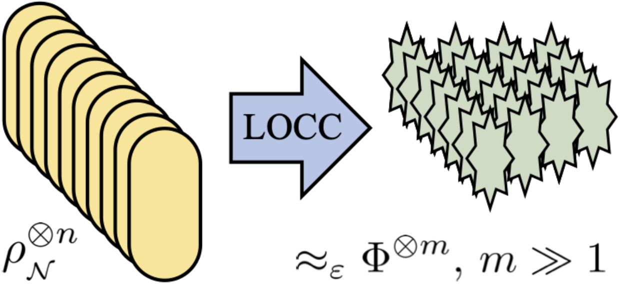
The Quantity Picture

Traditionally: Maximize the number of transmitted qubits



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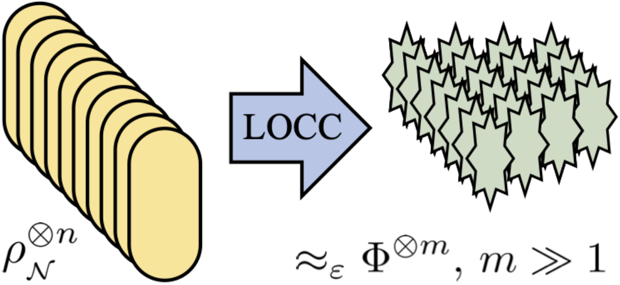

Goal: Characterize the optimal achievable rate m/n

Quantum Capacity

Two-Way Assisted Quantum Capacity:

$$Q_{\leftrightarrow}(\mathcal{N}) := \lim_{\varepsilon \to 0} \sup \left\{ \lim_{n\to\infty} \frac{m(\Lambda_n)}{n} : \rho(\Lambda_n) \approx_{\varepsilon} \Phi^{\otimes m},\ \Lambda_n \in \mathrm{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n}) \right\}$$

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$$Q_{\leftrightarrow}(\mathcal{N}) = E_{d,\leftrightarrow}(\rho_{\mathcal{N}})$$

with the distillable entanglement of the Choi state $E_{d,\leftrightarrow}(\rho_{\mathcal{N}})$.

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Best known upper-bound is the regularized REE:[2]

$$Q_{\leftrightarrow}(\mathcal{N}) \leq D^{\infty}(\rho \,\|\, \mathrm{SEP}) := \lim_{n\to\infty} \frac{1}{n} \inf_{\sigma_n \in \mathrm{SEP}} D\left(\rho^{\otimes n} \middle\| \sigma_n\right) \leq D(\rho \| \mathrm{SEP})$$

$\hookrightarrow$ Regularization is (1) generally necessary and (2) computationally very hard!

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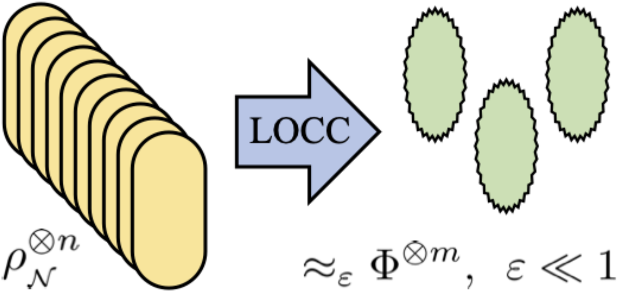
A Shift in Perspective

Today: Following [Lami/Berta/Regula, *Nat. Phys.* (2026)], maximise the quality of transmission.



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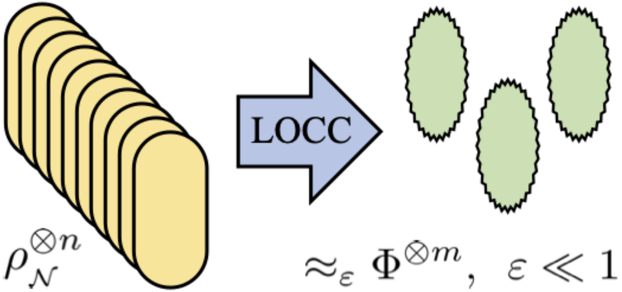

Goal: Characterize the optimal exponent c with $\varepsilon \sim 2^{-cn}$

Our guiding question

Can we give **quantitative bounds** on the **quality** of quantum communication and entanglement distribution over (very noisy) **optical channels** for which we do not have strong capacity characterisations?

First Main Finding: Yes!

Two-Way Assisted (Zero-Rate) Error Exponent:

$$Q_{\leftrightarrow,\mathrm{err}}(\mathcal{N}) = \lim_{m\to\infty} \sup \left\{ \lim_{n\to\infty} -\frac{1}{n} \log \varepsilon(\Lambda_n) : \rho(\Lambda_n) \approx_\varepsilon \Phi^{\otimes m}, \Lambda_n \in \mathrm{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n}) \right\}$$

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For teleportation-simulable channel $\mathcal{N}$ that acts on m bosonic modes:

$$Q_{\leftrightarrow,\mathrm{err}}(\mathcal{N}) \leq \liminf_{r\to\infty} D(\mathrm{SEP}\|\rho_{\mathcal{N}}(r)) =: E_{\mathrm{RR}}(\mathcal{N})$$

with the *quasi*-Choi state

$$\rho_{\mathcal{N}}(r) = (\mathcal{N} \otimes \mathcal{I})\big(\Phi(r)^{\otimes m}\big)\,,$$

where $\Phi(r)$ is two-mode squeezed vacuum state (with squeezing r).

$\hookrightarrow$ Crucially, regularisation is *superfluous* as $D(\mathrm{SEP}\|\rho)$ is additive!

Proof Ingredients

For any achievable error exponent s:

- Use twirling of the output to restrict to protocols with isotropic output

$$\rho\left(\Lambda_n^{(m)}\right) = f_n \Phi^{\otimes m} + (1 - f_n)\tau_{2^m} \quad \text{with} \quad f_n \geq 1 - 2^{-ns}$$

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- By teleportation-simulability $\rho\left(\Lambda_n^{(m)}\right) = \lim_{r\to\infty} \overline{\Lambda}_{n,r}^{(m)}\left(\rho_{\mathcal{N}}(r)^{\otimes n}\right)$:

$$\begin{aligned} D\left(\mathrm{SEP}\middle\|\rho\left(\Lambda_n^{(m)}\right)\right) &\leq \liminf_{r\to\infty} D\left(\mathrm{SEP}\middle\|\overline{\Lambda}_{n,r}^{(m)}\left(\rho_{\mathcal{N}}(r)^{\otimes n}\right)\right) && (1)\\ &\leq \liminf_{r\to\infty} D\left(\mathrm{SEP}\middle\|\rho_{\mathcal{N}}(r)^{\otimes n}\right) && (2)\\ &= n \liminf_{r\to\infty} D\left(\mathrm{SEP}\|\rho_{\mathcal{N}}(r)\right) && (3)\end{aligned}$$

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- By isotropic symmetry of the output:

$$D\left(\mathrm{SEP}\middle\|\rho\left(\Lambda_n^{(m)}\right)\right) = D_{\mathrm{bin}}(2^{-m}\|f_n) \geq -1 + ns\left(1-2^{-m}\right)$$

Second Main Finding

We have a **single-letter** upper bound on the quality of quantum communication, but is it actually **computable**?

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Yes! For Gaussian states ρ_G with covariance matrix $\boldsymbol{V}_\rho$:

$$D(\mathrm{SEP}\|\rho_G) = \min_{\boldsymbol{V}_\sigma, \gamma_A > 0} \quad \frac{\mathrm{tr}\{\boldsymbol{V}_\sigma(\boldsymbol{G}[\boldsymbol{V}_\rho] - \boldsymbol{G}[\boldsymbol{V}_\sigma])\}}{2\ln(2)} + \log_2 \sqrt{\frac{\det(\boldsymbol{V}_\rho + i\boldsymbol{\Omega}_{AB})}{\det(\boldsymbol{V}_\sigma + i\boldsymbol{\Omega}_{AB})}}$$

$$\text{s.t.} \quad \boldsymbol{V}_\sigma \geq \gamma_A \oplus i\boldsymbol{\Omega}_B \quad \text{and} \quad \gamma_A \geq i\boldsymbol{\Omega}_A$$

with the symplectic form $\boldsymbol{\Omega}$ and Gibbs matrix $\boldsymbol{G}[\boldsymbol{V}]$.

$\hookrightarrow$ Finite-dimensional convex program with two simple PSD constraints!

Proof Ingredients

Key Observation: For Gaussian states ρ_G: $D(\mathrm{SEP}\|\rho_G) = D(\mathrm{SEP}_G\|\rho_G)$

$\hookrightarrow$ It is strongly believed that this does not hold for $D(\rho_G\|\mathrm{SEP})$!

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Let σ_G be the *Gaussification* of σ (i.e. Gaussian state with same first and second moments):

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The convex program combines (1) separability criterion from [Lami/Serafini/Adesso, *New J. Phys.* (2018)] with (2) the statistical moment characterization of the relative entropy from [Pirandola *et al.*, Nat. Com. (2017)].

Example: Thermal Attenuator (1/2)

Analytical solutions available for the most important one-mode Gaussian channels, i.e. thermal attenuator and amplifier, and additive-Gaussian noise

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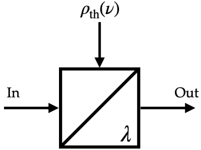
Thermal attenuator channel $\mathcal{N}_{\lambda,\nu}$ with transmissivity $\lambda \in [0, 1]$ and added thermal noise $\nu \in [0, \infty)$



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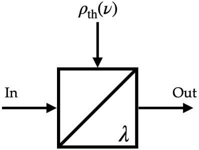


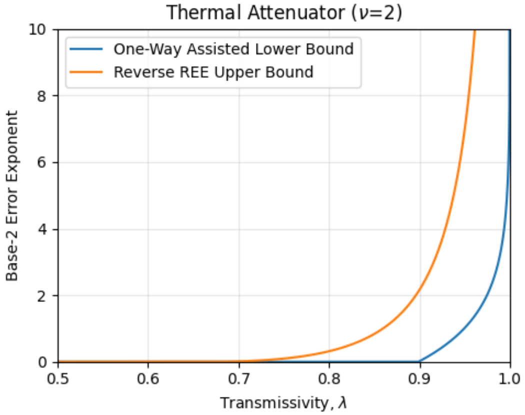
The (zero-rate) error exponent of the thermal attenuator is upper-bounded by

$$E_{\mathrm{RR}}\left(\mathcal{N}_{\lambda,\nu}\right) = \log_2\left[(1+\nu)(1-\lambda)\right] + \frac{\lambda}{1-\lambda}\log_2\left(\frac{\lambda}{\lambda_{\min}}\right) \text{ with } \lambda_{\min} := \frac{\nu}{\nu+1}$$

for $\lambda \in [\lambda_{\min}, 1]$ and zero otherwise.

Example: Thermal Attenuator (2/2)

Achievability Question: Lifted *one-way assisted* random-coding-based bound from [Berta *et al.*, arXiv:2602.17430] to one-mode Gaussian channels

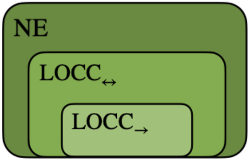


Bonus: Operational Interpretation

Exact operational interpretation in entanglement distillation with *non-entangling* operations:

$$D(\mathrm{SEP}\|\rho) = E_{d,\mathrm{err}}^{\mathrm{NE}}(\rho)\,,$$

where $E_{d,\mathrm{err}}^{\mathrm{NE}}(\rho_{AB})$ is (zero-rate) error exponent of entanglement distillation .



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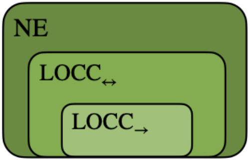

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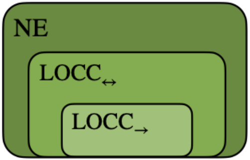

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The reverse relative entropy of entanglement is thus the first *operational* entanglement measure that is also *efficiently computable* for Gaussian states.

Conclusion

Takehome Message:
The quality perspective provides a promising alternative to study optical quantum communication, circumventing many roadblocks of the capacity framework.

Some future directions:

- Improvement of achievability result?
- Extensions to other (Gaussian) resource theories?

Based on arXiv:2510.07121, please see for details the full paper: